A discussion on dynamical systems, time-periodic mechanical systems, and Floquet theory.

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Abstract

Coupled systems of second order differential equations naturally arise in applied mechanics and mechanical engineering as the equations of motion for a mechanical device. For many engineering applications, as the avoidance of instabilities is sought, after a suitable canonical choice of coordinates and possible linearization around a central equilibrium, Lypunouv stability is determined via the eigenvalue of the Jacobian with maximal real part. This spectral stability test is well ingrained in the engineering industry with many commercial software packages focused on efficiently applying the technique computationally to systems of large degree. However in practise, application of the technique is often applied näively without due consideration for the limiting criteria.

This presentation will address second order linear differential equations with time-periodic coefficients which often arise as the equations of motion for systems involving, e.g. rotors. Paradigmatic examples have been studied in-depth by the dynamics and vibrations group of TU Darmstadt over several years and some on-going projects will be summarised. It will be stressed that the above stability test is *not* applicable in these cases and emphasised that Floquet theory is the appropriate approach to determine stability. In this case, time-periodicity is removed by a suitable coordinate transformation as specified by Floquet, with the eigenvalue of the monodromy matrix with largest absolute value, known as the maximal Floquet multiplier, determining stability.

As a parameter is varied, e.g. the rotor speed, a bifurcation occurs and the system becomes unstable if a Floquet multiplier crosses the unit circle. Qualitatively different transitory dynamics results depending on whether the critical multiplier occurs at 1, -1, a multiple of 1, or as part of a complex conjugate pair. In addition, symmetries in the equation of motion can limit the type of bifurcations permitted and thereby the resultant dynamics.

A robust and efficient technique for calculating the Floquet multipliers numerically is therefore vital to understand realistic engineering models. Large stiff systems cause particular problems in obtaining reliable and accurate approximations of the monodromy matrix. For example, the computational cost of the direct integration method grows exponentially with size of the system as one is required to integrate over the period for each degree of freedom. Other, less well known, techniques exist which can make some computational savings by skipping monodromy matrix calculation and obtaining Floquet multipliers indirectly.

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