Robust Damping in Self-Excited Mechanical Systems

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Outline

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   - Destabilizing effect of damping

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   - Minimal model of disc brake
   - Optimization results

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Self-excited mechanical systems
Introduction

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- non-linear equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} = \mathbf{f}_{nl}(t, \mathbf{q}, \dot{\mathbf{q}}), \qquad \mathbf{q} \in \mathbb{R}^n$$

- linearization around $\mathbf{q} \equiv \mathbf{0}$

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D} + \mathbf{G})\dot{\mathbf{q}} + (\mathbf{K} + \mathbf{N})\mathbf{q} = \mathbf{0}$$

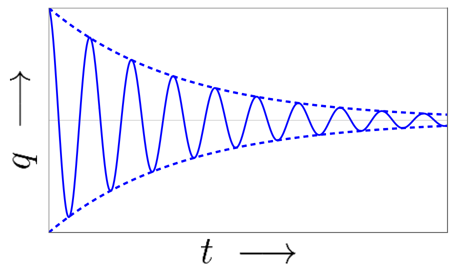
$\mathbf{M} = \mathbf{M}^{\mathrm{T}}$	inertia
$\mathbf{D} = \mathbf{D}^{\mathrm{T}}$	damping
$\mathbf{G} = -\mathbf{G}^{\mathrm{T}}$	gyroscopic
$\mathbf{K} = \mathbf{K}^{\mathrm{T}}$	stiffness
$\mathbf{N} = -\mathbf{N}^{\mathrm{T}}$	circulatory

- the ansatz $\mathbf{q} = \mathbf{v}e^{\lambda t}$ leads to the eigenvalue problem

$$[\mathbf{M}\lambda^2 + (\mathbf{D} + \mathbf{G})\lambda + \mathbf{K} + \mathbf{N}]\mathbf{v} = \mathbf{0}$$

- asymptotic stability

$$\mathrm{Re}(\lambda) < 0 \qquad \forall \lambda \in \mathbb{C}$$



Self-excited mechanical systems
Examples

- ground resonance in helicopters [1]
- vibrations in paper calenders [2]
- brake squeal [3]
  - friction-induced sound
  - pure comfort problem in most cases
  - does not affect main function

Damping in circulatory systems
A simple example

- linear equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{q}} + (\mathbf{K}+\mathbf{N})\mathbf{q} = \mathbf{0}, \qquad \mathbf{q} = (q_1, q_2)^{\mathrm{T}}$$

with

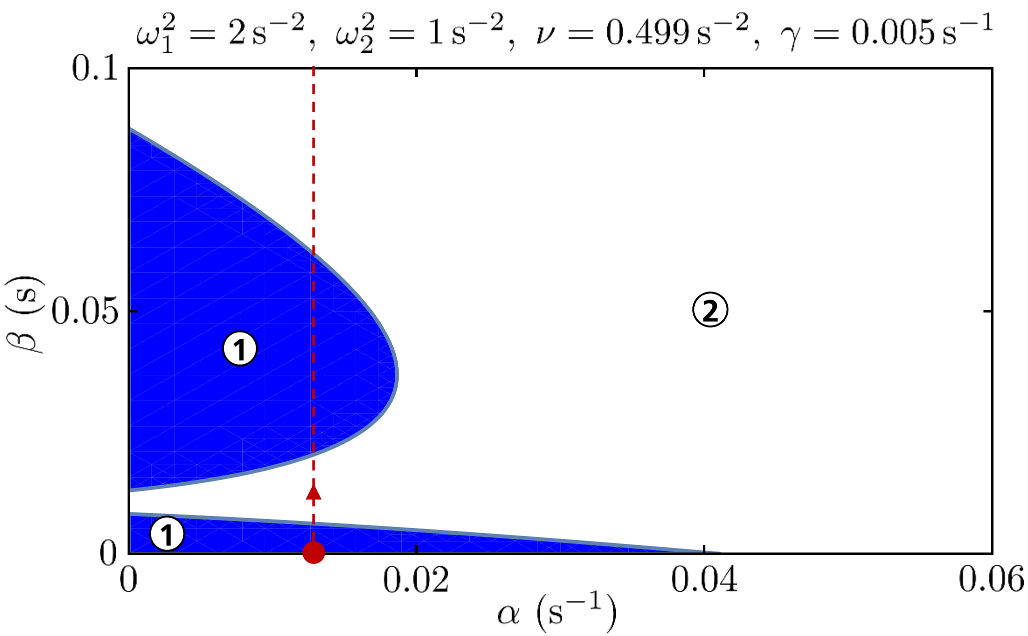
$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \mathbf{G} = \begin{pmatrix} 0 & \gamma \\ -\gamma & 0 \end{pmatrix}, \; \mathbf{K} = \begin{pmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{pmatrix}, \; \mathbf{N} = \begin{pmatrix} 0 & \nu \\ -\nu & 0 \end{pmatrix}$$

and Rayleigh damping

$$\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K}, \qquad \alpha, \beta \in \mathbb{R}$$

Damping in circulatory systems
A simple example

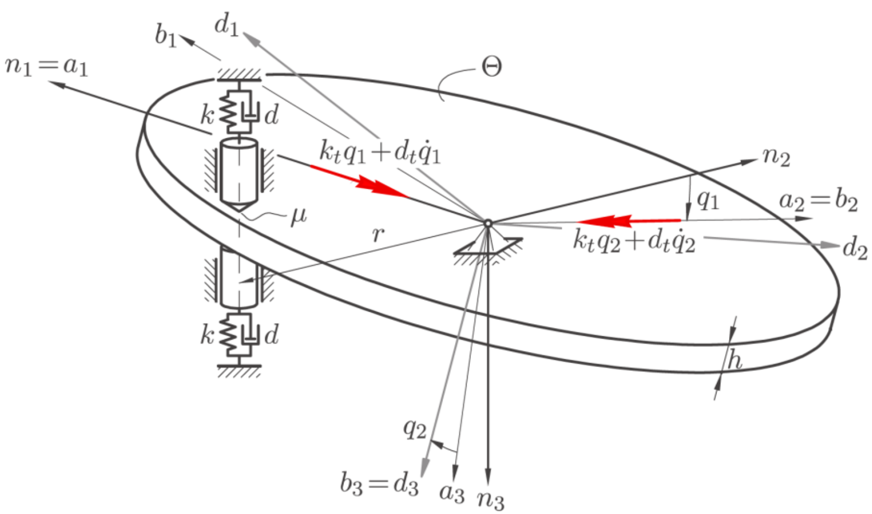
stability map in α-β-plane



① flutter instability ② asymptotic stability

- for $\alpha < 0.02\ \mathrm{s}^{-1}$, the stability of the trivial solution strongly depends on the value for β
- for $\alpha > 0.04\ \mathrm{s}^{-1}$, the system will be asymptotically stable for any value of β
- the effect of the damping matrix significantly depends on its structure

Minimal model of wobbling disc brake
Introduction



von Wagner et al. 2007 [4]

- rigid disc rotating with constant angular speed hinged in a spherical joint
- two guided pins in frictional contact with the disc
- assumptions:
  - COULOMB's law
  - no sticking → equations of motion can be linearized

Minimal model of wobbling disc brake
Parameter reduction

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- equations of motion from Ref. [4]

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{q}} + (\mathbf{K}+\mathbf{N})\mathbf{q} = \mathbf{0}, \qquad \mathbf{q} = (q_1, q_2)^{\mathrm{T}}$$

- rescaling the vector of generalized coordinates and the time

$$\mathbf{q} = q_c\tilde{\mathbf{q}}, \qquad \tilde{\mathbf{q}} = (\tilde{q}_1, \tilde{q}_2)^{\mathrm{T}}$$

$$t = t_c\tau$$

$$q_c, t_c = \mathrm{const.}$$

- rescaled equations of motion

$$\tilde{\mathbf{M}}\tilde{\mathbf{q}}'' + (\tilde{\mathbf{D}}+\tilde{\mathbf{G}})\tilde{\mathbf{q}}' + (\tilde{\mathbf{K}}+\tilde{\mathbf{N}})\tilde{\mathbf{q}} = \mathbf{0}, \qquad \frac{d(\cdot)}{d\tau} = (\cdot)'$$

- reduction from 10 to 7 dimensionless parameters

Minimal model of wobbling disc brake
Dimensionless matrices

$$\tilde{\mathbf{M}} = \begin{pmatrix} \tilde{\Theta} & 0 \\ 0 & \tilde{\Theta} \end{pmatrix}$$

$$\tilde{\mathbf{D}} = \begin{pmatrix} \tilde{d}_t + 2\tilde{d} + \frac{1}{2}\mu l^2 & -\frac{1}{2}\mu\tilde{d}l \\ -\frac{1}{2}\mu\tilde{d}l & \tilde{d}_t \end{pmatrix}$$

$$\tilde{\mathbf{G}} = \begin{pmatrix} 0 & 2\tilde{\Theta} + \frac{1}{2}\mu\tilde{d}l \\ -(2\tilde{\Theta} + \frac{1}{2}\mu\tilde{d}l) & 0 \end{pmatrix}$$

$$\tilde{\mathbf{K}} = \begin{pmatrix} \tilde{k}_t + 2\tilde{k} + l & -\frac{1}{4}\mu[(2\tilde{k} - l)l + 4] \\ -\frac{1}{4}\mu[(2\tilde{k} - l)l + 4] & \tilde{k}_t + (1 + \mu^2)l \end{pmatrix}$$

$$\tilde{\mathbf{N}} = \begin{pmatrix} 0 & \frac{1}{4}\mu[(2\tilde{k} + l)l + 4] \\ -\frac{1}{4}\mu[(2\tilde{k} + l)l + 4] & 0 \end{pmatrix}$$

$$\tilde{\Theta} = \frac{\Theta\Omega^2}{N_0 r} \approx 0.101$$

$$l = \frac{h}{r} \approx 0.154$$

$$\mu = 0.6$$

$$\tilde{k} = \frac{kr}{N_0} = 260$$

$$\tilde{k}_t = \frac{k_t}{N_0 r} \approx 48205$$

$$\tilde{d} = \frac{dr\Omega}{N_0} \approx 3.40 \cdot 10^{-3}$$

$$\tilde{d}_t = \frac{d_t\Omega}{N_0 r} \approx 4.03 \cdot 10^{-3}$$

Minimal model of wobbling disc brake
Decomposition of $\widetilde{\mathbf{D}}$

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- Dimensionless damping matrix

$$\tilde{\mathbf{D}} = \tilde{\mathbf{D}}_0 + \sum_{j=1}^{k} \alpha_j \tilde{\mathbf{D}}_j \qquad \alpha_j > 0$$

$$= \underbrace{\begin{pmatrix} \frac{1}{2}\mu l^2 & -\frac{1}{2}\mu\tilde{d}l \\ -\frac{1}{2}\mu\tilde{d}l & 0 \end{pmatrix}}_{\text{Coulomb damping}} + \alpha_1 \underbrace{\begin{pmatrix} \tilde{d}_t & 0 \\ 0 & \tilde{d}_t \end{pmatrix}}_{\text{disc}} + \alpha_2 \underbrace{\begin{pmatrix} 2\tilde{d} & 0 \\ 0 & 0 \end{pmatrix}}_{\text{pins}}$$

$\tilde{d}_t$ damping in the disc

$\tilde{d}$ damping in the pins

μ friction coefficient

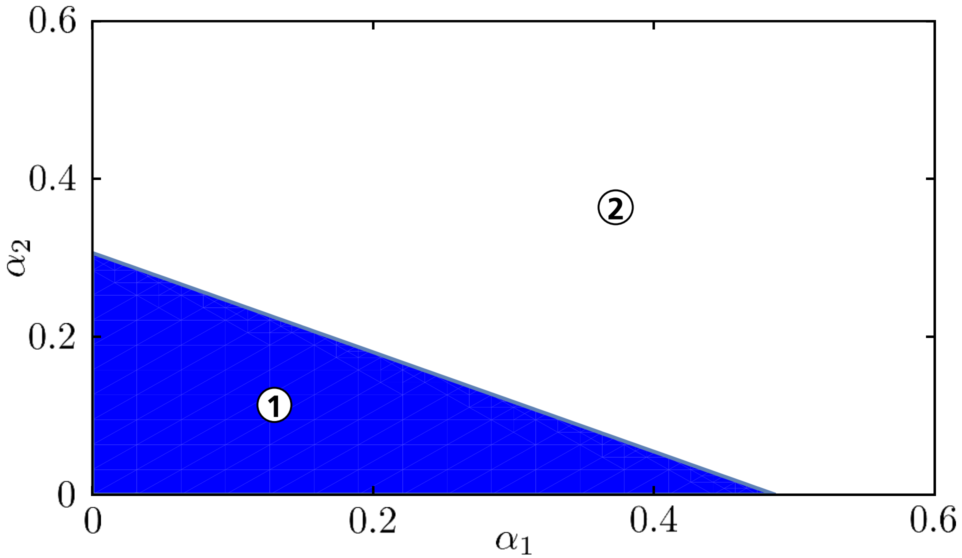
l thickness-radius ratio

- eigenvalues of $\widetilde{\mathbf{M}}\widetilde{\mathbf{D}}\widetilde{\mathbf{G}}\widetilde{\mathbf{K}}\widetilde{\mathbf{N}}$-system

$$\lambda_i = \lambda_i(\alpha_1, \alpha_2, \ldots, \alpha_k) \qquad i = 1, 2, \ldots, 2n$$

Minimal model of wobbling disc brake
Stability considerations

stability map in α_1-α_2-plane

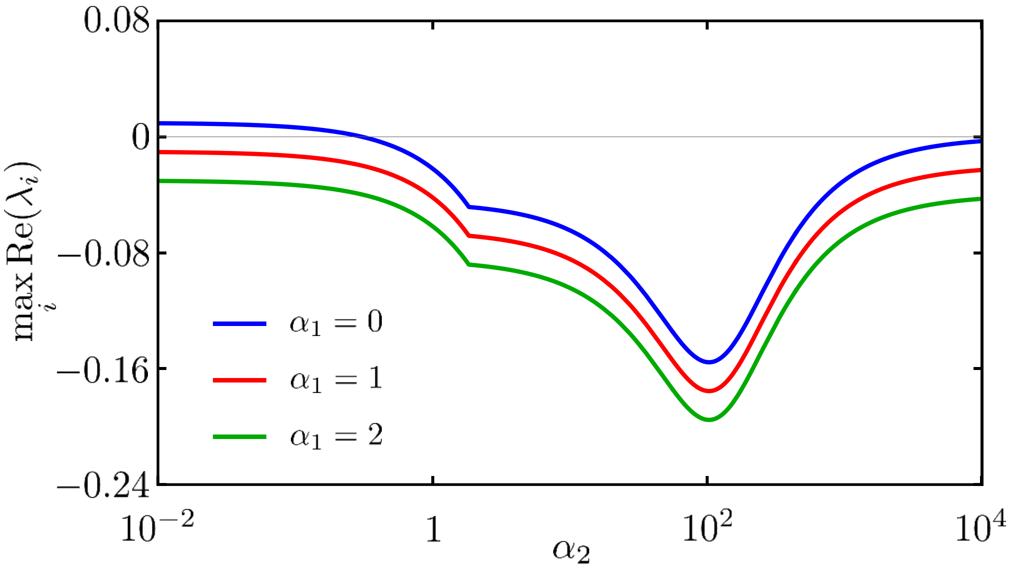


① flutter instability ② asymptotic stability

- the trivial solution is unstable if only COULOMB damping is active ($\alpha_1 = \alpha_2 = 0$)
- stabilization by adding sufficiently high damping in the disc and in the pins
- optimum point for α_1 and α_2?

Minimal model of wobbling disc brake
Stability considerations

maximum real part over α_2



- increasing α_1 (damping in the disc) always stabilizes
- the optimum value for α_2 (damping in the pins) is $\alpha_{2opt} \approx 100$
- optimum point in technically relevant range?

Minimal model of wobbling disc brake
Optimization of $\widetilde{\mathrm{D}}$

- optimization problem

$$\min_{\alpha_j} \max_i \mathrm{Re}(\lambda_i)$$
$$\text{s.t.}$$
$$\sum_{j=1}^{k} \alpha_j = k$$ → constant total damping $\alpha_1 + \alpha_2 = 2$
$$\alpha_{min} \leq \alpha_j \leq \alpha_{max}$$ → upper and lower limits $0.1 \leq \alpha_j \leq 2$

- optimization by Mathematica function „NMinimize”
  - constrained nonlinear optimization
  - Nelder-Mead method

Minimal model of wobbling disc brake
Optimization of $\widetilde{\mathrm{D}}$

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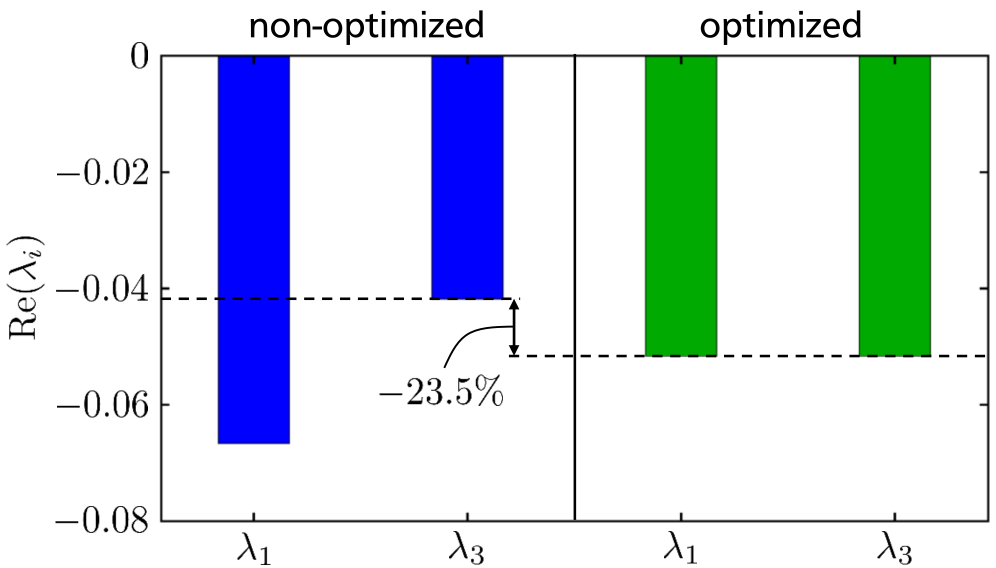
optimization results



- the maximum real part can be reduced by 23.5 %
- the equilibrium state is more stable with the optimized structure of the damping matrix

Minimal model of wobbling disc brake
Optimization of $\widetilde{\mathbf{D}}$

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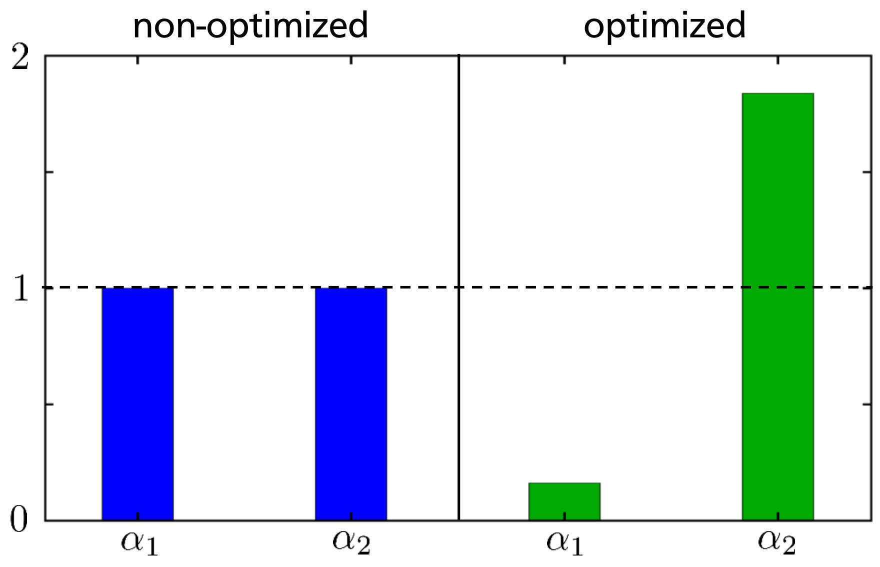
optimization results



$$\tilde{\mathbf{D}} = \tilde{\mathbf{D}}_0 + \alpha_1 \underbrace{\begin{pmatrix} \tilde{d}_t & 0 \\ 0 & \tilde{d}_t \end{pmatrix}}_{\text{disc}} + \alpha_2 \underbrace{\begin{pmatrix} 2\tilde{d} & 0 \\ 0 & 0 \end{pmatrix}}_{\text{pins}}$$

- recommendation of the optimization:
  - damping in the disc ↓
  - damping in the pins ↑

Summary and outlook

results:

- the decomposition of the damping matrix is also applicable to larger matrices
- the stability boundary can be adjusted by scaling the different damping matrices
- the optimization of the structure of the damping matrix stabilizes the equilibrium state

still do be done:

- the transfer of the found results to larger models

References

[1] E. J. Clerkin, and A. Karev. *Modelling of self-excited vibrations in time-variant mechanical systems*. Ongoing DFG project, DFG HA 1060/56-1.

[2] M. Eckstein. *Instabilities and wear propagation in calenders. Interactions with structural dynamics and contact kinematics*. PhD thesis, TU Darmstadt, 2014.

[3] A. Wagner. *Avoidance of brake squeal by a separation of the brake disc's eigenfrequencies. A structural optimization problem.* PhD thesis, TU Darmstadt, 2013.

[4] U. von Wagner, D. Hochlenert. P. Hagedorn, *Minimal Models for Disk Brake Squeal*, J. Sound Vib. 302 (3), 527–539 (2007).