Robust Damping in Self-Excited Mechanical Systems

Dominic Jekel, Eoin Clerkin, and Peter Hagedorn
TU Darmstadt, fnb, Dynamics and Vibrations Group

December 18, 2015

In mechanical engineering there are many systems exhibiting self-excited vibrations, e.g. squealing disc brakes, the ground resonance phenomenon in helicopters, and the vibrations in paper calenders. Although these are nonlinear systems, a standard stability analysis approach is to linearize the equations of motion around an equilibrium position. For self-excited systems, in the linearized equations of motion, non-conservative, circulatory forces are present, leading to skew-symmetric matrices in the coordinate-proportional forces. It is well known, that such systems are very sensitive to damping and that their stability strongly depends on the structure of the damping matrix [1, 2]. Therefore, the main goal of this contribution is to optimize the structure of the damping matrix in order to stabilize a general mechanical system. After linearization, the following equations of motion are obtained:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + (\boldsymbol{D} + \boldsymbol{G})\dot{\boldsymbol{q}} + (\boldsymbol{K} + \boldsymbol{N})\boldsymbol{q} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{q}$ is the column vector of the generalized coordinates. The mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$ are taken to be symmetric and positive definite. The gyroscopic matrix $\boldsymbol{G}$ and the circulatory matrix $\boldsymbol{N}$ are skew-symmetric, while the damping matrix $\boldsymbol{D}$ is symmetric. The matrices are square of size n, where n for many studies can be large. For the optimization problem and to account for the different physical origins of damping, $\boldsymbol{D}$ can be decomposed into

$$\boldsymbol{D} = \boldsymbol{D_0} + \sum_{j=1}^{k} \alpha_j \boldsymbol{D}_j, \tag{2}$$

with the global damping matrix $\boldsymbol{D_0}$ and k additional α_j-weighted damping matrices $\boldsymbol{D}_j$, where each $\boldsymbol{D}_j$ is due to a different damping component. In case of the minimal model of the wobbling disc brake presented in Ref. [3], damping in the disc and in the pins, as well as damping due to dry friction can expressed by equation (2).

All relevant physical parameters of system (1) are assumed to be known, while the weighting factors α_j are unknown. Using standard complex eigenvalue analysis, the stability ($\mathrm{Re}(\lambda_i) < 0$) of the equilibrium state can be investigated as a function of the weighting factors α_j. Hence, it is possible to adjust the stability boundary by scaling the additional damping matrices $\boldsymbol{D}_j$. An optimization problem over the α_j can be formulated, such that the real part of the maximal eigenvalue is minimized. The weighting factors α_j are varied between industry obtained upper and lower limits.

In this presentation, we investigate mechanical models with two degrees of freedom. In this particular case, due to the small number of parameters, the influence of the different damping terms on the stability behaviour of system (1) can be completely understood.

References

[1] P. Hagedorn, M. Eckstein, E. Heffel, and A. Wagner: Self-Excited Vibrations and Damping in Circulatory Systems. Journal of Applied Mechanics 81 (10): 101009-1–9, 2014.

[2] P. C. Müller: Stabilität und Matrizen. Matrizenverfahren in der Stabilitätstheorie linearer dynamischer Systeme. Springer, Berlin, Heidelberg, New York, 1977.

[3] U. von Wagner, D. Hochlenert, and P. Hagedorn: Minimal Models for Disk Brake Squeal. Journal of Sound and Vibration 302 (3): 527–539, 2007.