Dynamics of coupled Single Mode Lasers

E.Clerkin[1,2], S.O'Brien[1], E.P.O'Reilly[1,2], A.Amann[1,3]

[1] *Tyndall National Institute, Lee Maltings, Prospect Row, Cork*
[2] *Physics Department, University College Cork*
[3] *School of Mathematical Sciences, University College Cork*

Tyndall National Institute
Institiúid Náisiúnta

Bifurcation Theory

Consider a system of autonomous ordinary differential equations (ODEs)

$$\dot{x} = F\left(x\ ,\mu\right)$$

where x and μ are an array of variables and parameters. F are the corresponding functions.

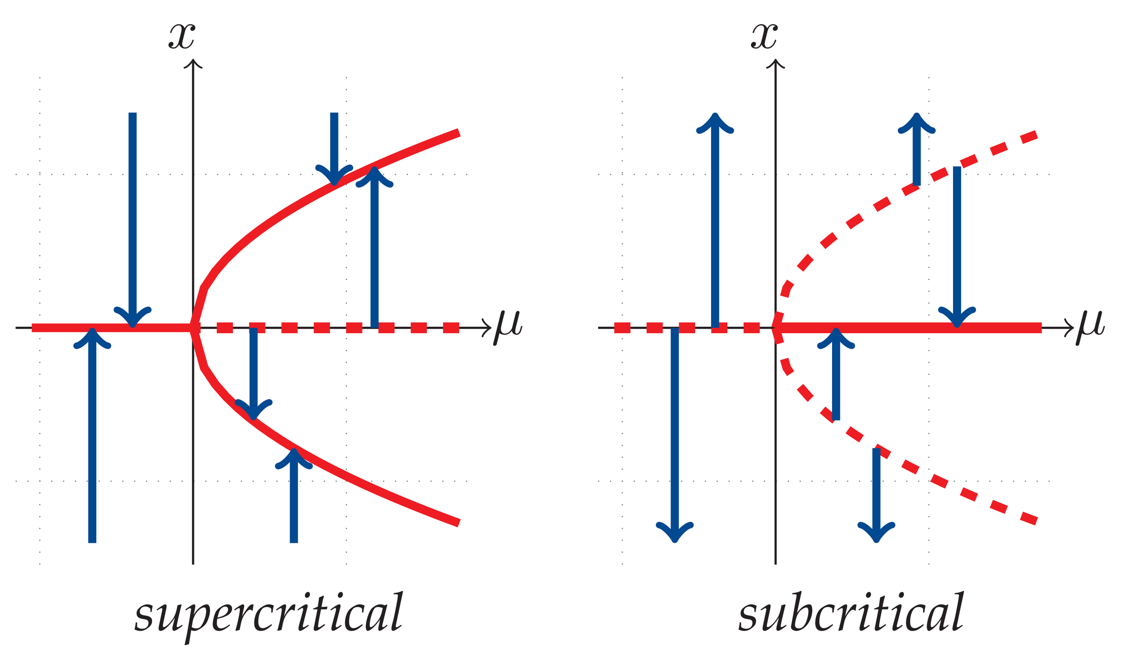
Definition A point x_0 with parameter μ_0 is a **equilbria** of the system if $F\left(x_0, \mu_0\right) = 0$.

Implicit Function Theorem Given a continuous function $F : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ such that $F(x_0,\mu_0) = 0$, $\exists$ open ngbds $A \subset \mathbb{R}^n$, $B \subset \mathbb{R}^m$ of x_0,μ_0. $\forall\mu \in B, F(\bullet,\mu) : A \rightarrow \mathbb{R}^n$ is locally one-to-one then $\exists$ open ngbd $A_0 \subset \mathbb{R}^n$ and $B_0 \subset \mathbb{R}^m$ of x_0 and μ_0, such that $\forall\mu \in B_0$, the equation $F(x_0, \mu_0) = 0$ has a unique solution $x = g(\mu) \in A_0$.

Definition A point $(x_0$,$\mu_0)$ is a **bifurcation** point if several branches of fixed points come together.

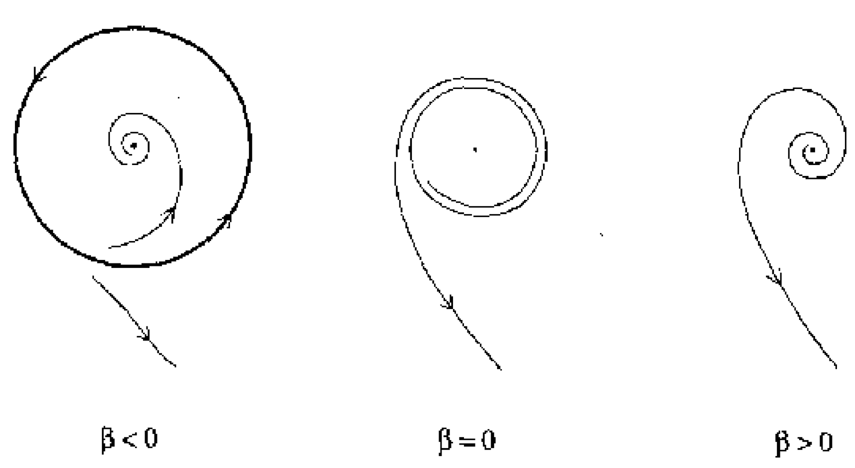
Two simple yet important bifurcations.

i) Pitchforks $\dot{x} = x(\mu - x^2),\ x \in \mathbb{R}$



detection: $\partial_x^3 F(x_0,\mu_0) \neq 0$,$\partial_{\mu x}^2 F(x_0,\mu_0) \neq 0$
$\partial_x F(x_0,\mu_0) = 0, \partial_x^2 F(x_0,\mu_0) = 0, \partial_\mu F(x_0,\mu_0) = 0$

ii) Hopfs $\dot{z} = z(\lambda - b|z|^2),\ \lambda < 0,\ z, b = \beta + i\gamma \in \mathbb{C}$



detection: The real part of a complex conjugate pair of eigenvalues of the Jacobian of a fixed point crosses through zero.

References

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[2] R.Lang, K. Kobayashi *External optical feedback effects on semiconductor injection laser properties.* IEEE J. Quantum Electron., (1980) pp. 347-353

[3] S.Yanchuk, K.R.Schneider, L.Recke *Dynamics of two mutually coupled semiconductor lasers: Instataneous coupling limit* Physical review E6, 056221 (2004)

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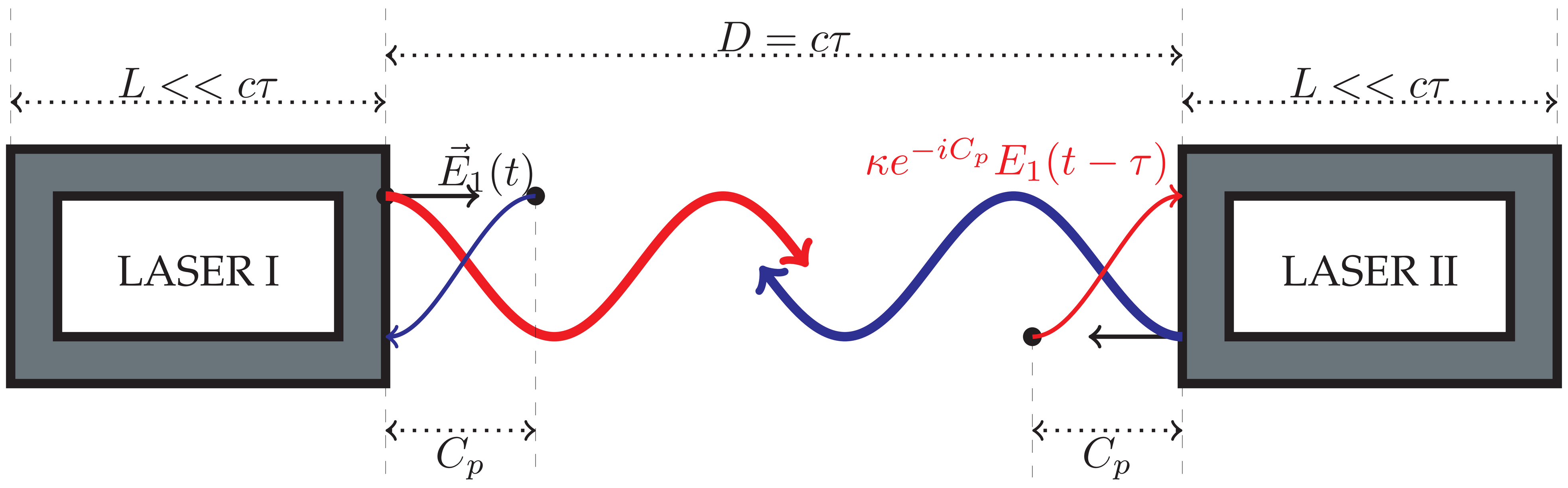
Laser Rate Equations



Laser systems have proved themselves a playground for the non-linear theorist displaying some of the richest dynamics in physics. The mutually coupled lasers sketched above, can be modeled by a modified version of the Lang-Kobayashi equations [2].

$$\begin{aligned}
\dot{E}_1(t) &= (1+i\alpha)N_1(t)E_1(t) + \kappa e^{-iC_p}E_2(t-\tau) - i\Delta E_1(t) \\
\dot{E}_2(t) &= (1+i\alpha)N_2(t)E_2(t) + \kappa e^{-iC_p}E_1(t-\tau) + i\Delta E_2(t) \\
T\dot{N}_1(t) &= P - N_1(t) - (1+2N_1(t))|E_1(t)|^2 \\
T\dot{N}_2(t) &= P - N_2(t) - (1+2N_2(t))|E_2(t)|^2
\end{aligned}$$

variables: E = complex electric fields, N = carrier densities.

C_p	phase of injected light
κ	amount of injected light
τ	delay
Δ	relative detuning
T	photon/carrier lifetime
α	line enhancement factor
P	pumping

Stable Dynamics Map

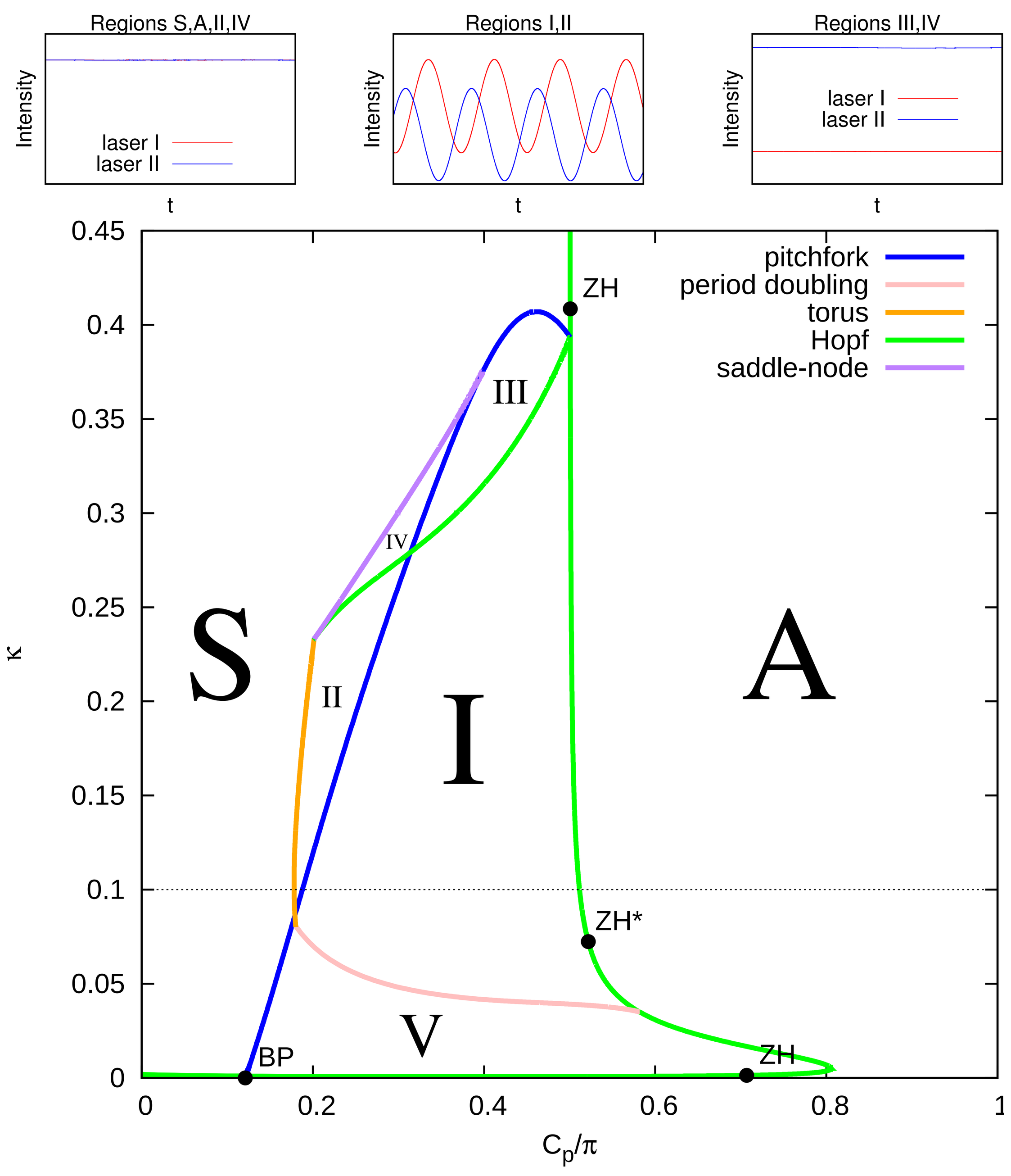
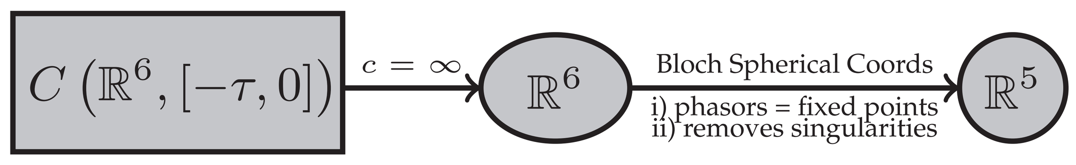


Figure 3: A C_p and κ parameter map displaying the character of dynamics for small delayed coupled laser systems. See small time traces above the main map and title for different regions. Here S labels in-phase sychronous and A anti-phase synchronous CW output. Rest are labeled via roman numerials. Region V was not the focus of this study. Parameters chosen are $T = 392, \alpha = 2.5, \Delta = 0$

Coordinate Change



Three new variables instead of the electric fields

$$q_x + iq_y = 2E_1^* E_2$$
$$q_z = |E_1|^2 - |E_2|^2$$

Writing the inversions as their sum N_S and difference N_D, the system simplifys greatly

$$\begin{aligned}
\dot{q_x} &= q_x N_S - \alpha q_y N_D - 2\Delta q_y + 2\kappa R\cos(C_p) \\
\dot{q_y} &= q_y N_S + \alpha q_x N_D + 2\Delta q_x - 2\kappa q_z \sin(C_p) \\
\dot{q_z} &= q_z N_S - R N_D + 2\kappa q_y \sin(C_p) \\
T\dot{N_D} &= [q_z(1+N_S) - (R+1)N_D] \\
T\dot{N_S} &= [2P - (R+1)N_S - R - q_z N_D]
\end{aligned}$$

where $R = \sqrt{q_x^2 + q_y^2 + q_z^2} = |E_1|^2 + |E_2|^2$

Some Symmetries

$$(C_p) \rightarrow (C_p + 2\pi)$$
$$(C_p, \kappa) \rightarrow (C_p + \pi, -\kappa)$$
$$(q_x, q_y, C_p) \rightarrow (-q_x, -q_y, C_p + \pi)$$
$$(q_y, q_z, N_1, N_2, \Delta) \rightarrow (-q_y, -q_z, N_2, N_1, -\Delta)$$

Connections between Fixed Points

κ=0.1

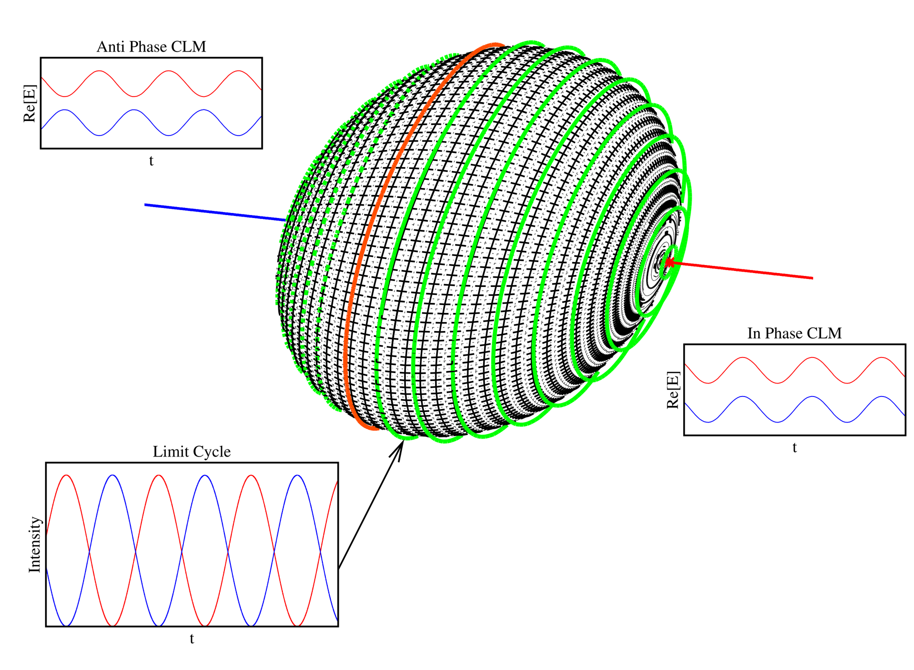


Figure 1: In phase (red) and anti phase (blue) synchronous solution are connected after Hopf bifurcations via limit cycles (green) which scan out and define an almost perfect Bloch sphere in these natural coordinates. Bottom inset shows the problem point for other coordinate systems where a stable orbit exists where each laser in turn momentarily turns off. Subtle difference between in-phase and anti-phase states are emphaised by the two small insets.

Figure 2: Synchronous solutions are confined to the qx axis. In phase (red) and anti-phase (blue) are connected via mixed phase states (purple) after two subcritical pitchfork bifurcations (P). Limit cycle (green) also connection the two synchronous states (red and blue) via Hopf bifurcations (H). These limit cycles exchange stability via a supercritical pitchfork of limit cycles (PL and orange in opposite graph) which gives birth to two bi-stable cycles.