Robust Damping in Self-Excited Mechanical Systems

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A technique to optimize the stability of a general mechanical system is outlined. The method relies on decomposing the damping matrix into several component matrices, which may have some special structure or physical relevance. An optimization problem can then be formulated where the ratio of these are varied to either stabilize or make more stable the equilibrium state subject to sensible constraints. For the purpose of this study, we define a system to be more stable if its eigenvalue with largest real part is as negative as possible. The technique is demonstrated by applying it to an introduced non-dimensionalized variant of a known minimal wobbling disc brake model. In this case, it is shown to be beneficial to shift some damping from the disc to the pins for a system optimized for stability.

1 General mechanical systems and a wobbling disc brake model

In mechanical engineering, there are many systems exhibiting self-excited vibrations, e.g. squealing disc brakes, the ground resonance phenomenon in helicopters, and the vibrations in paper calenders. It is well known that such systems are very sensitive to damping and that their stability may strongly depend on the structure of the damping matrix [1–3]. Although the problems are inherently nonlinear, a standard stability analysis approach involves linearizing the equations of motion around an equilibrium which results in a coupled system of second order linear differential equations of the form

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D} + \mathbf{G})\dot{\mathbf{q}} + (\mathbf{K} + \mathbf{N})\mathbf{q} = \mathbf{0}, \tag{1}$$

where $\mathbf{q}$ is the column vector of the generalized coordinates. The mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ are symmetric and positive definite, while the gyroscopic matrix $\mathbf{G}$ and the circulatory matrix $\mathbf{N}$ are skew-symmetric. The damping matrix $\mathbf{D}$ is symmetric and is the focus of this study. Explanation of differentiation between matrices can be found in Ref. [4].

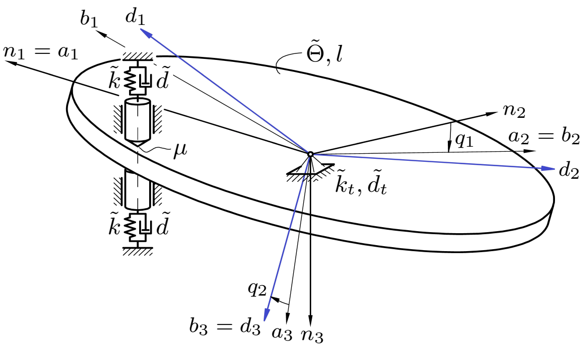

Fig. 1: Minimal model of wobbling disc brake, Ref. [5], which along with Tab. 1 defines Eqs. (1) and (2).

Table 1: Non-dimensionalized parameters based on those of Ref. [5].

Notational symbol	Numerical value	Ref. [5] relation	Parameter description
μ	0.6	μ	friction coefficient
l	0.154	$\frac{h}{r}$	thickness to radius ratio
$\tilde{\Theta}$	0.101	$\frac{\Theta\Omega^2}{N_0 r}$	mass moment of inertia
$\tilde{k}$	260	$\frac{kr}{N_0}$	pin stiffness
$\tilde{k}_t$	48205	$\frac{k_t}{N_0 r}$	rotational stiffness
$\tilde{d}$	3.40×10^{-3}	$\frac{dr\Omega}{N_0}$	pin damping
$\tilde{d}_t$	4.03×10^{-3}	$\frac{d_t\Omega}{N_0 r}$	disc damping

For industrially relevant applications, the methods used in this investigation must apply to large matrices. To demonstrate the technique, a minimal model of a wobbling disc brake with two degrees of freedom is analyzed. As shown in Fig. 1, a rigid disc is hinged with its centre of mass supported by rotational springs and dampers such that it can perform wobbling motions described by the angles q_1 and q_2. The disc is in frictional contact with two guided pins with their own stiffness and damping properties. The model and equations of motion are derived in Ref. [5]. To reduce the number of parameters and to compare the damping and stiffness coefficients of the disc and the pins, non-dimensionalization is conducted on these equations. This results in the following dimensionless system matrices:

$$\begin{gathered}\mathbf{M} = \begin{pmatrix} \tilde{\Theta} & 0 \\ 0 & \tilde{\Theta} \end{pmatrix}, \ \mathbf{D} = \begin{pmatrix} \tilde{d}_t + 2\tilde{d} + \frac{1}{2}\mu l^2 & -\frac{1}{2}\mu\tilde{d}l \\ -\frac{1}{2}\mu\tilde{d}l & \tilde{d}_t \end{pmatrix}, \ \mathbf{G} = \begin{pmatrix} 0 & 2\tilde{\Theta} + \frac{1}{2}\mu\tilde{d}l \\ -(2\tilde{\Theta} + \frac{1}{2}\mu\tilde{d}l) & 0 \end{pmatrix}, \\ \mathbf{K} = \begin{pmatrix} \tilde{k}_t + 2\tilde{k} + l & -\frac{1}{4}\mu[(2\tilde{k} - l)l + 4] \\ -\frac{1}{4}\mu[(2\tilde{k} - l)l + 4] & \tilde{k}_t + (1 + \mu^2)l \end{pmatrix}, \ \mathbf{N} = \begin{pmatrix} 0 & \frac{1}{4}\mu[(2\tilde{k} + l)l + 4] \\ -\frac{1}{4}\mu[(2\tilde{k} + l)l + 4] & 0 \end{pmatrix}.\end{gathered} \tag{2}$$

Parameter values are chosen to compare with Ref. [5], with Tab. 1 showing their relation between the two papers.

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2 A formulation of an optimization problem

For the formulation of an optimization problem and to account for the different physical origins, the damping matrix $\mathbf{D}$ of Eqs. (2) can be written as a linear combination

$$\mathbf{D} = \mathbf{D}_0 + \sum_{j=1}^{k} \alpha_j \mathbf{D}_j = \underbrace{\begin{pmatrix} \frac{1}{2}\mu l^2 & -\frac{1}{2}\mu \tilde{d} l \\ -\frac{1}{2}\mu \tilde{d} l & 0 \end{pmatrix}}_{\text{COULOMB damping}} + \alpha_1 \underbrace{\begin{pmatrix} \tilde{d}_t & 0 \\ 0 & \tilde{d}_t \end{pmatrix}}_{\text{disc damping}} + \alpha_2 \underbrace{\begin{pmatrix} 2\tilde{d} & 0 \\ 0 & 0 \end{pmatrix}}_{\text{pin damping}}, \tag{3}$$

where $\mathbf{D}_0$ is a matrix containing fixed damping terms and $\mathbf{D}_j$ are α_j-weighted damping matrices whose relative contribution to the overall damping is to be varied. Thus, for the wobbling disc brake model presented, damping due to the disc, damping due to the pins, and damping due to COULOMB friction can be treated independently. Using standard complex eigenvalue analysis and a form of the damping matrix according to Eq. (3), the stability of system (1) can be optimized by varying the weighting factors α_j. For this purpose, a system is considered to be more stable if its eigenvalue with largest real part is as negative as possible. In general, problems of this form can be mathematically expressed as

$$\min_{\alpha_j} \max_{i} \operatorname{Re}(\lambda_i) \qquad \text{subject to} \qquad \mathbf{g}(\alpha_j) = \mathbf{0} \quad \text{and} \quad \mathbf{h}(\alpha_j) \leq \mathbf{0}, \tag{4}$$

where $\mathbf{g}$ and $\mathbf{h}$ are equality and inequality constraints, respectively. In the specified example, the optimization method will involve varying the ratio of the damping due to the disc and the damping due to the pins. Hence, it is convenient to write $\alpha_1 + \alpha_2 = 2$ with $\alpha_1 > 0$ and $\alpha_2 > 0$. Optimization of Eq. (4) is sought using the Nelder-Mead algorithm which is a simplex-based direct search method representing a suitable choice as it can optimize over multiple parameters. This is implemented in MATHEMATICA's `NMinimize` package.

3 Result of the optimization of the wobbling disc brake

For the system defined by Eqs. (1)–(3) and Tab. 1, the non-optimized state is stable as can be seen by the negative real parts of the eigenvalues in Fig. 2a where the initial weighting factors, $\alpha_1 = 1$ and $\alpha_2 = 1$, are shown in Fig. 3a. In the optimized case, the real parts of the eigenvalues become balanced, cf. Fig. 2b. Thus, we find the equilibrium state of system (1) is 24 % more stable in the sense of its least stable eigenvalue having a more negative real part. Fig. 3b shows that the corresponding weighting factor α_1 is reduced and α_2 is enlarged. With regard to stability and to the given constraints, it therefore can be concluded that it is beneficial to shift some damping from the disc to the pins. All methods and techniques used to optimize stability for the low dimensional wobbling disc brake example are scalable to large industrially relevant systems.

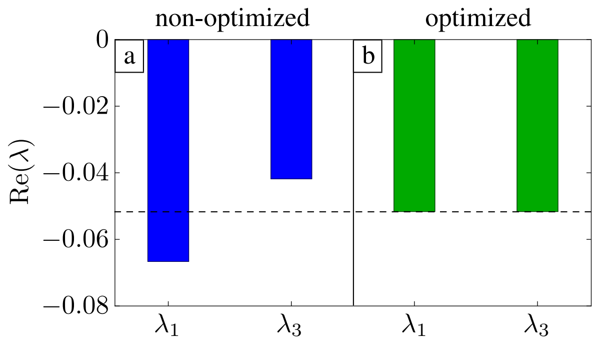

Fig. 2: Comparison of non-optimized and optimized real parts of the two complex conjugate pairs of eigenvalues.

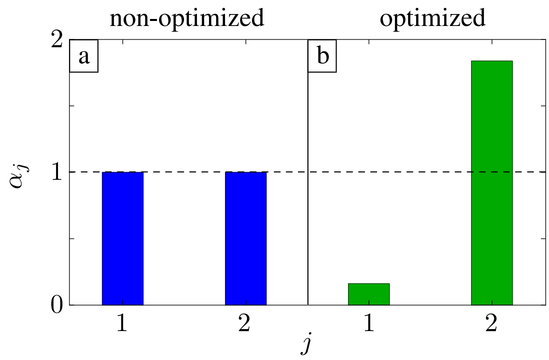

Fig. 3: Comparison of non-optimized and optimized α_j for the damping coefficient of the disc ($j = 1$) and the pins ($j = 2$).

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